



Thinking Mathematically: Searching for Patterns!

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What is Mathematics?

Print Gallery (M. C. Escher, 1956)

Science vs. Mathematics



SCIENCE

- **Goal:** Understand Nature
- **Method:** Experiments
- **Core Concept:** Falsifiable Hypothesis
 - *Make a guess, check if it's true!*

MATHEMATICS

- **Goal:** Understand Patterns
- **Method:** Proofs
- **Core Concept:** Consistent Axioms
 - *Set the rules, obey them!*

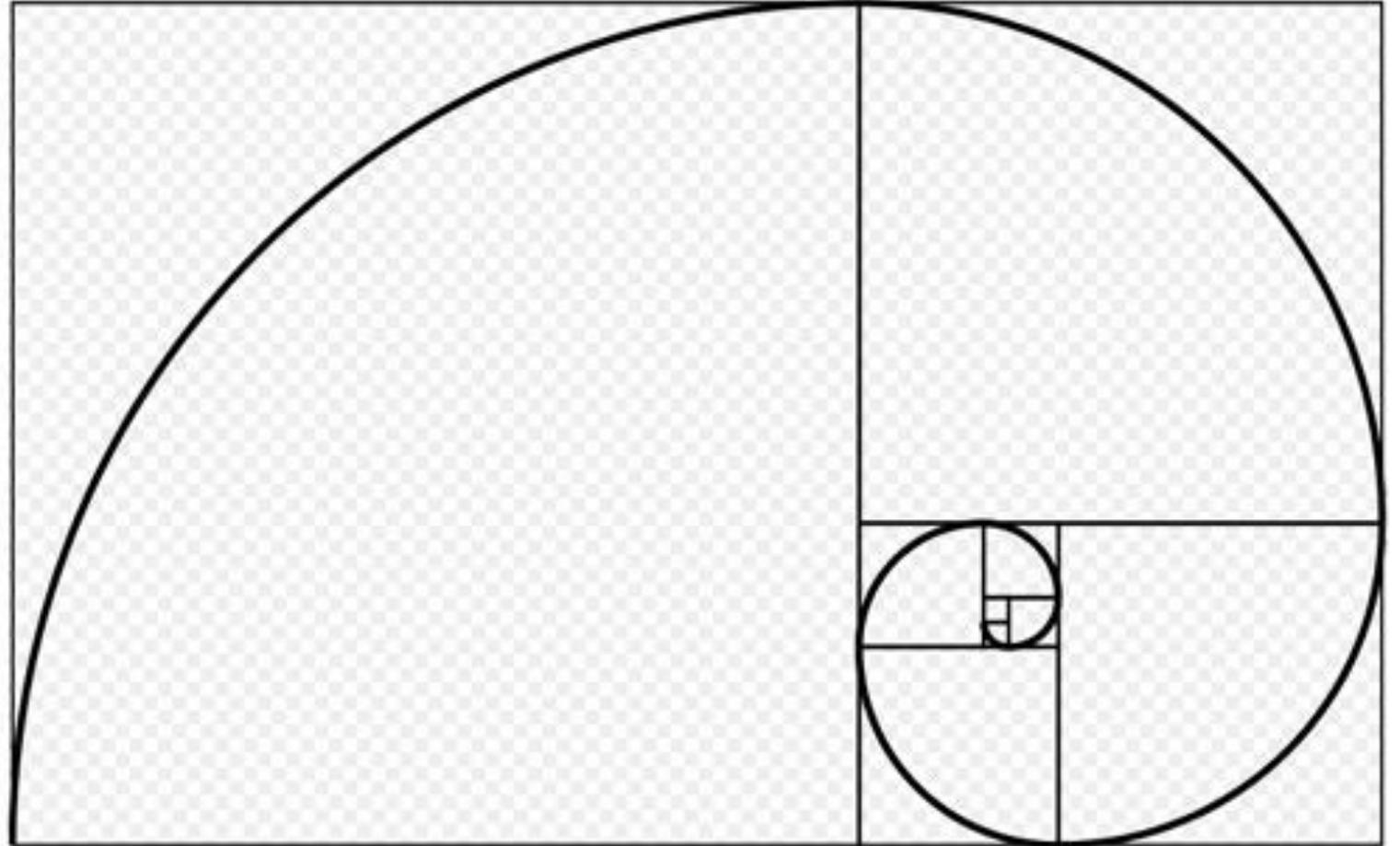


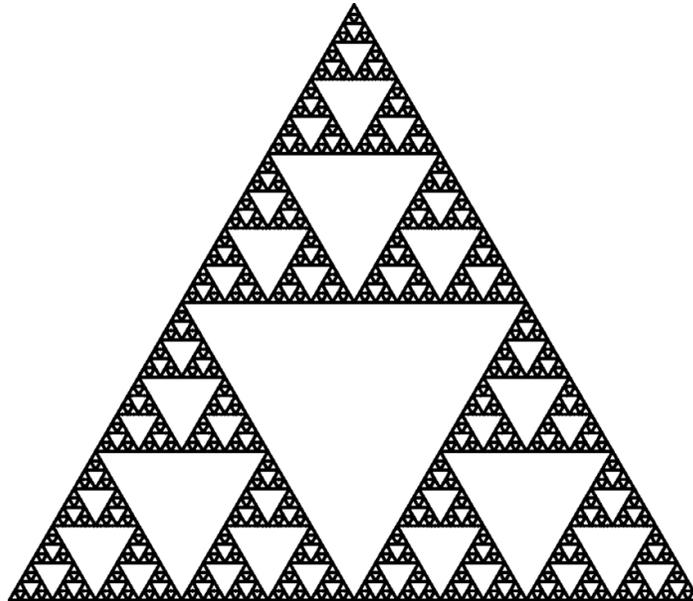
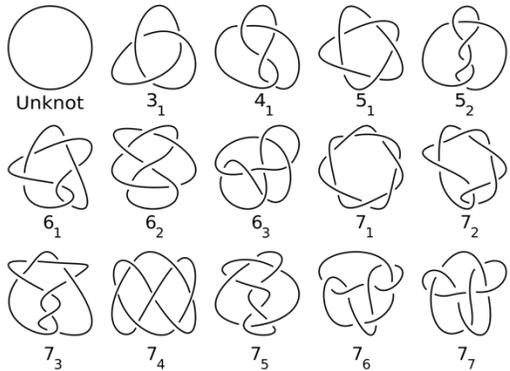
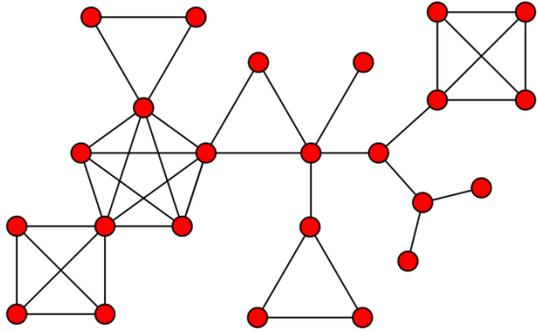
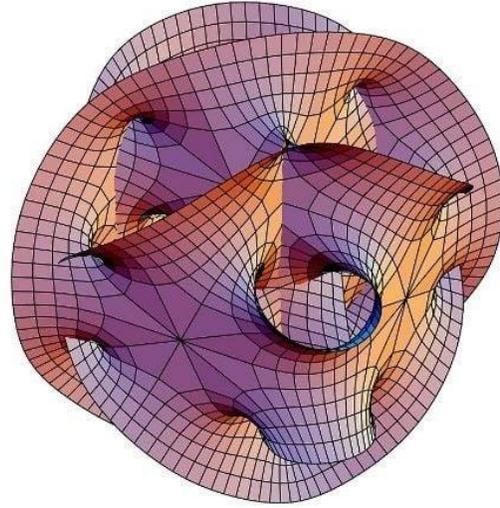
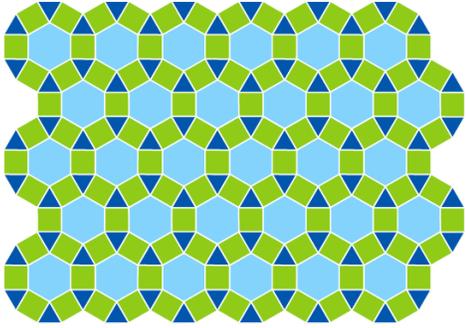
Patterns in Nature

YOU CANNOT DO SCIENCE
WITHOUT MATHEMATICS.

Abstract Patterns

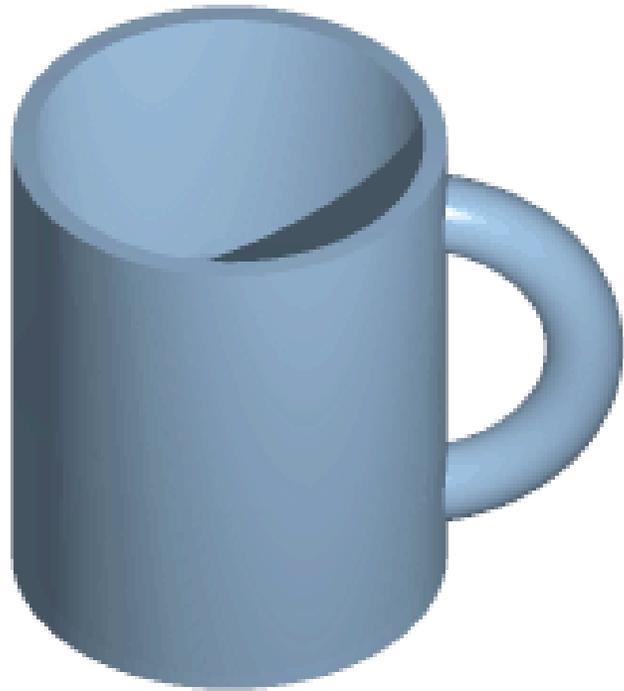
YOU CAN DO
MATHEMATICS
WITHOUT SCIENCE!





Some Interesting Mathematical Patterns

- Tilings
- Networks
- Knots
- Manifolds
- Fractals



Topology

- The study of *stretchy shapes*!
- Two shapes are the “same” if they can be stretched into each other.
- Count the number of “holes” in a shape.

How many “different” shapes in the alphabet?

A B C D E F

G H I J K L

M N O P Q

R S T U V

W X Y Z

What About Hiragana?

あ い う え お
か き く け こ
さ し す せ そ
た ち つ て と
な に ぬ ね の
は ひ ふ へ ほ
ま み む め も
や ゆ よ
ら り る れ ろ
わ を
ん

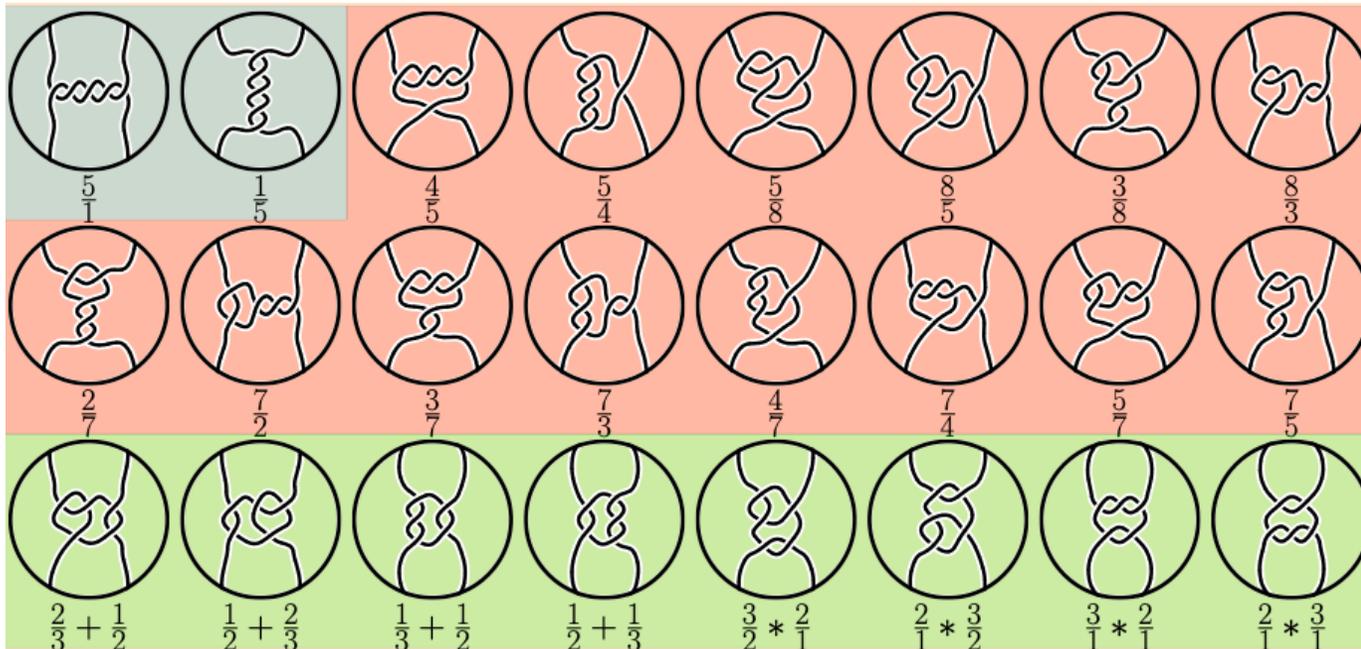


What About Kanji?

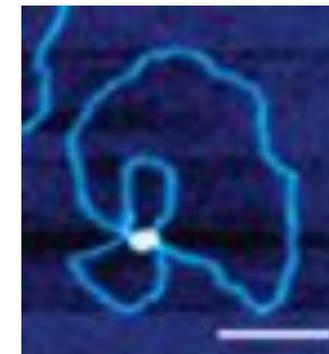
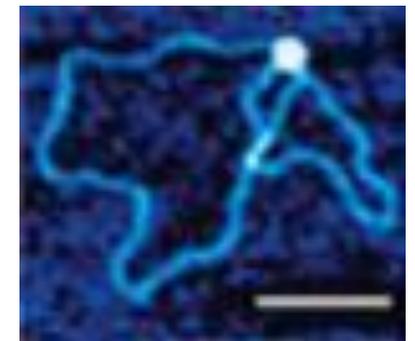
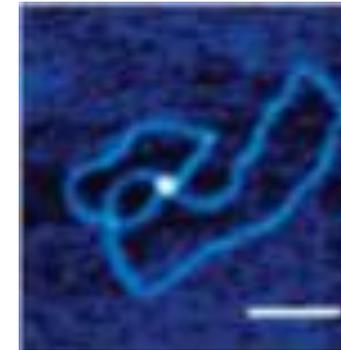
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音馱都漢着反切屋定体次
明旅区可品終紙部予身肉
赤広村題返始館園画育鳥
青店町宿払早別公計教魚
白林市不員起特全守顏菜
黒遠県意物欠凶引留頭野
色近国強事飯地答親苦理
花通界勉仕閉合場新薬料
回週世弱用開最工局楽米
茶道所考使号度受便病風
英馬信能番失者問声医取



Examples from Knot Theory

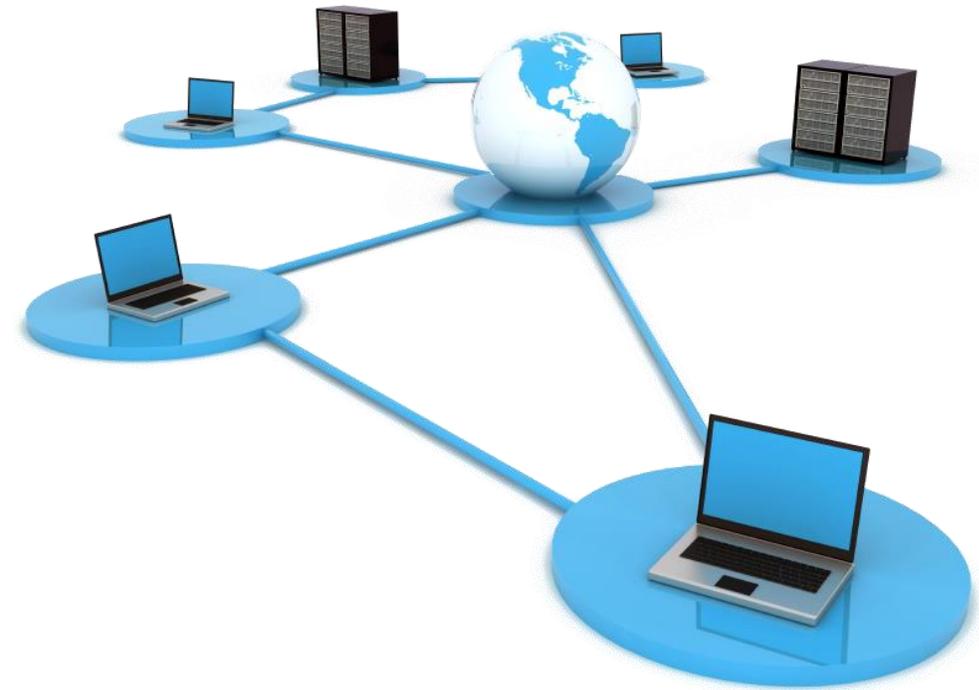
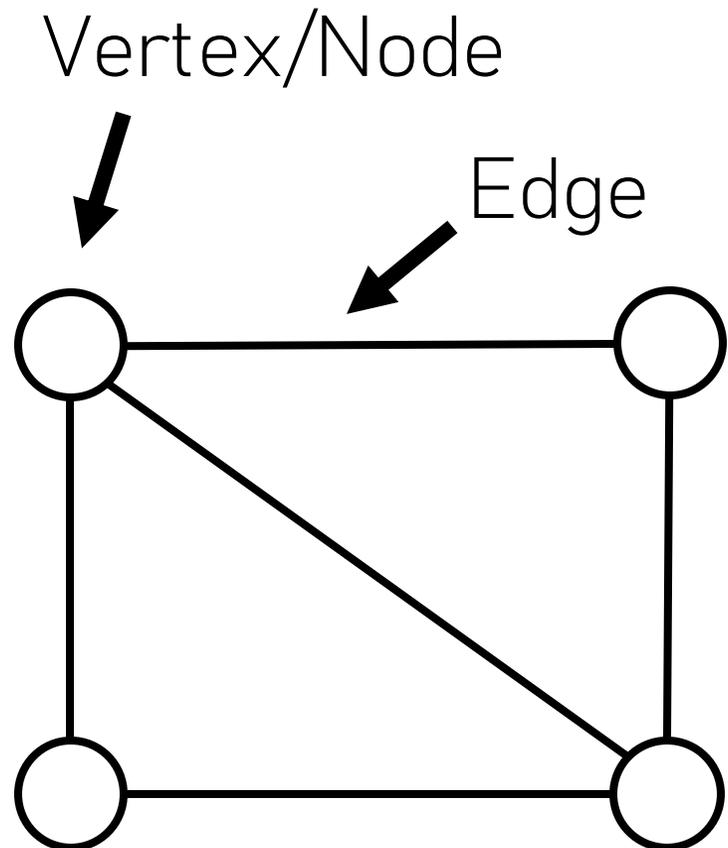


Connolly, Nicholas. *Classification and tabulation of 2-string tangles: The astronomy of subtangle decompositions*. The University of Iowa, 2021.



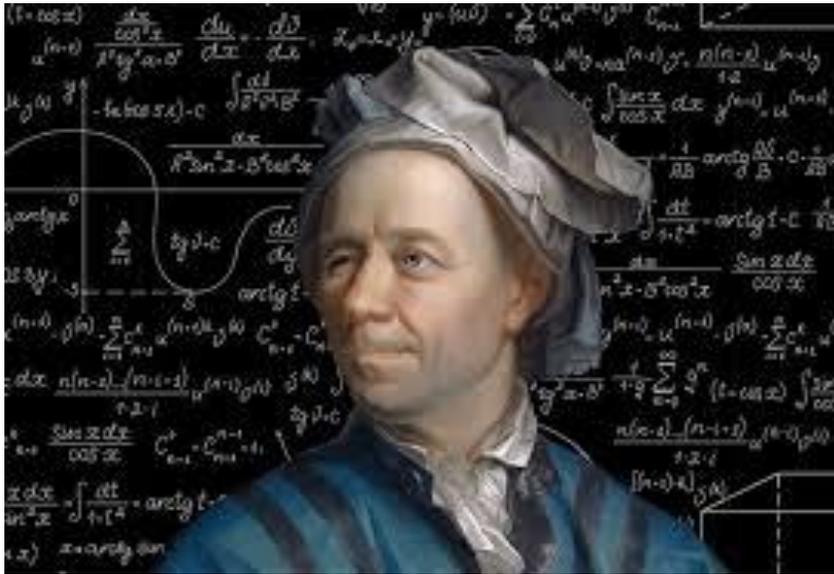
Vetcher, Alexandre A., et al. "DNA topology and geometry in Flp and Cre recombination." *Journal of molecular biology* 357.4 (2006).

Mathematical Graphs (Networks)

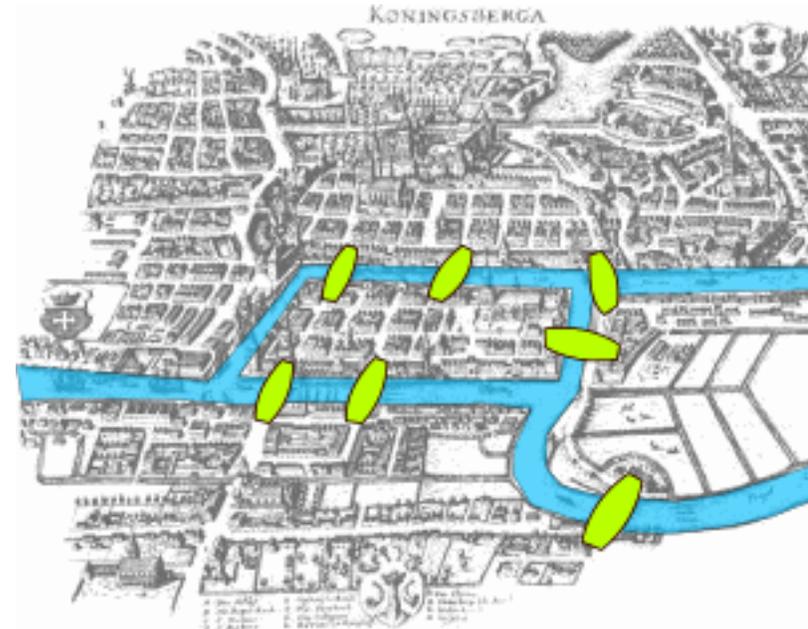


Example: the Internet!

The First Graph Theory Problem: The Seven Bridges of Königsberg

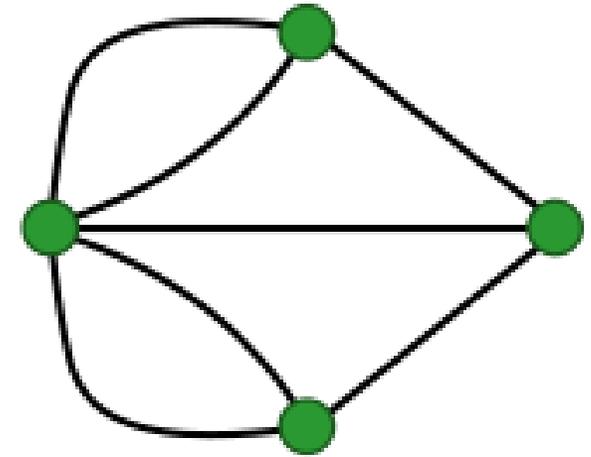
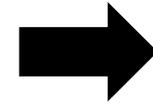
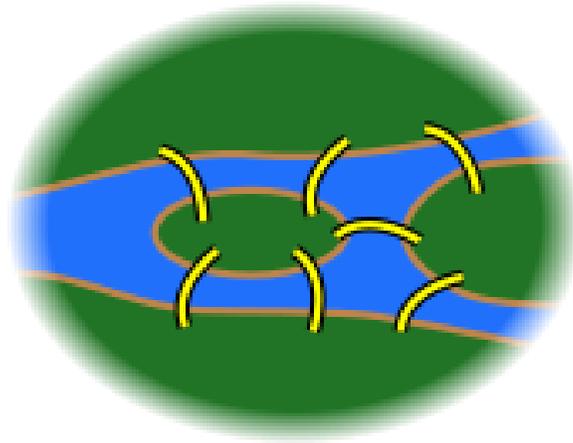
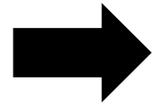
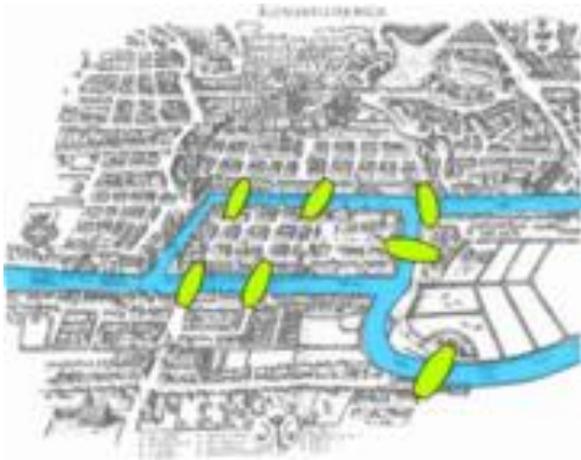


Leonhard Euler (1707)
Swiss Mathematician



City of Königsberg

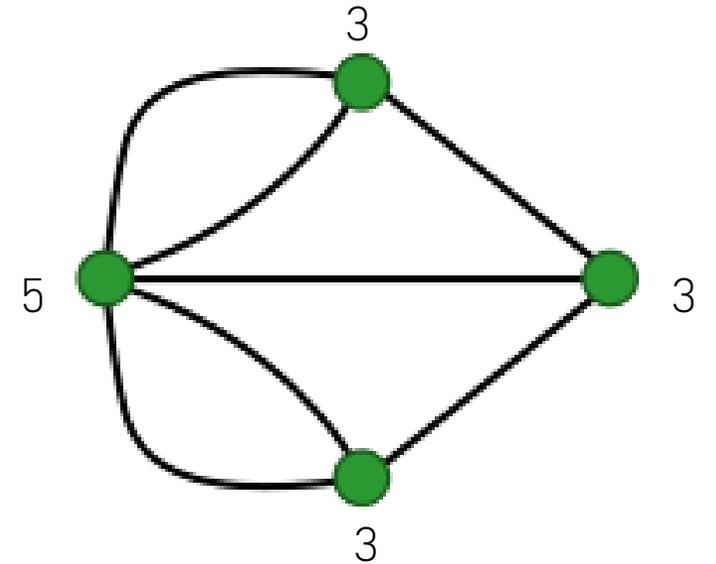
Representing Königsberg with a Graph



Is it possible to pass through each edge exactly once, without repeating the same edge twice?

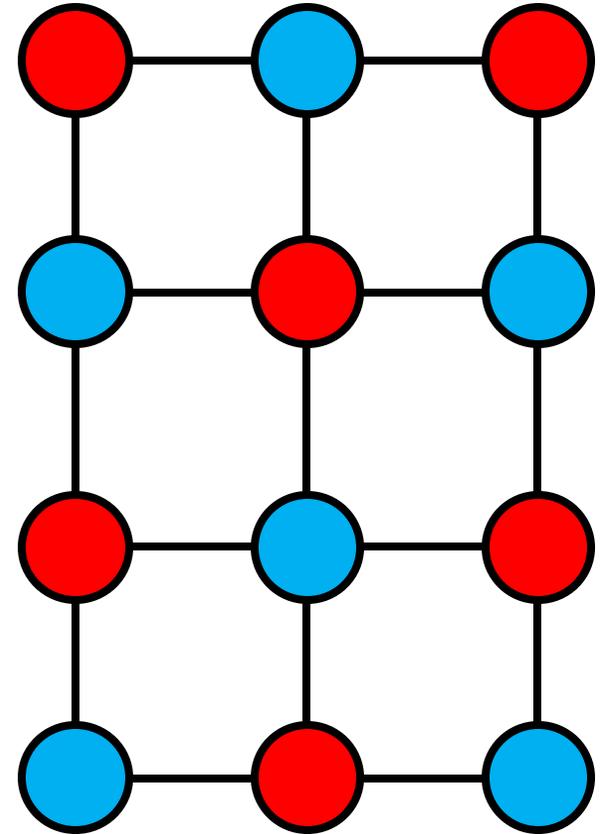
The Königsberg Solution

- The **degree** of a node is the number of connected edges.
- Each node in the Königsberg graph has *odd* degree.
- An **Euler trail** is a path through each edge *exactly once*.
- An Euler trail *only* exists if there are 0 or 2 odd degree nodes.
- No solution!



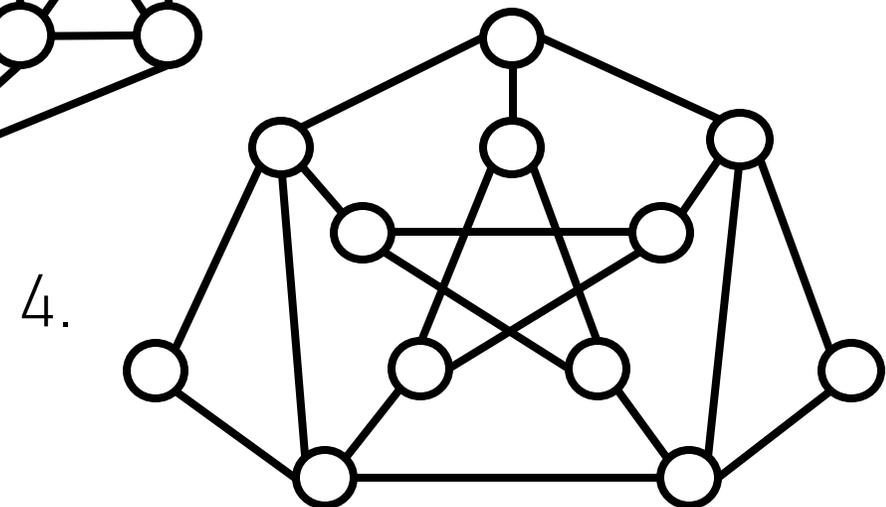
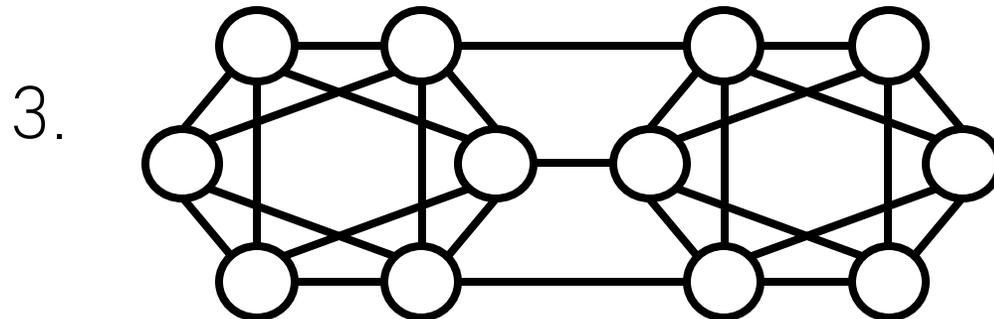
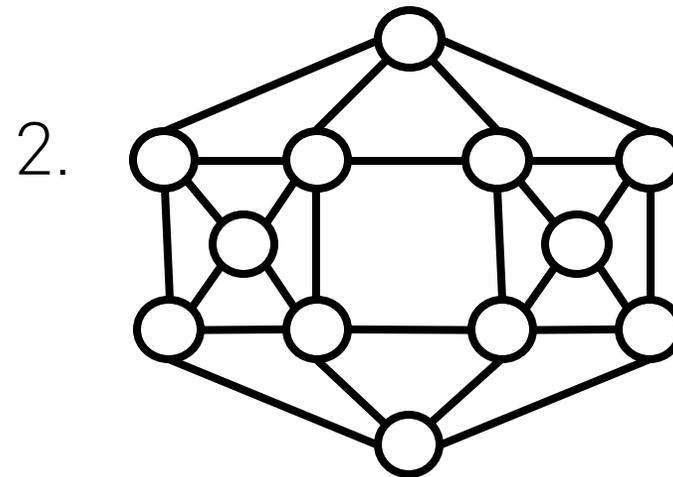
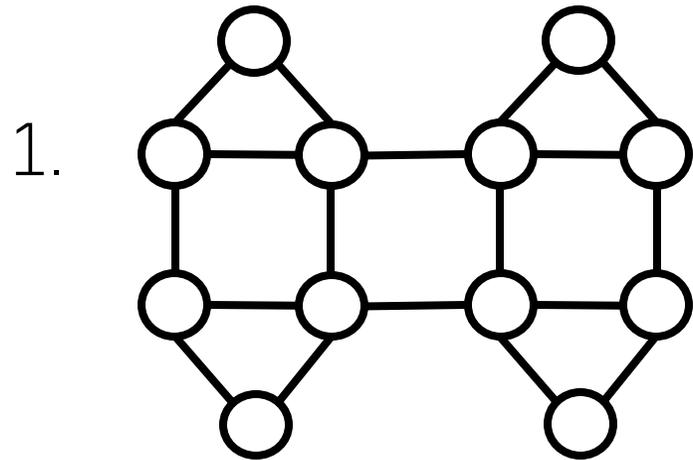
Graph Colorings

- A **coloring** of a graph follows these rules:
 - Choose a color for each node;
 - Connected nodes cannot use the same color.

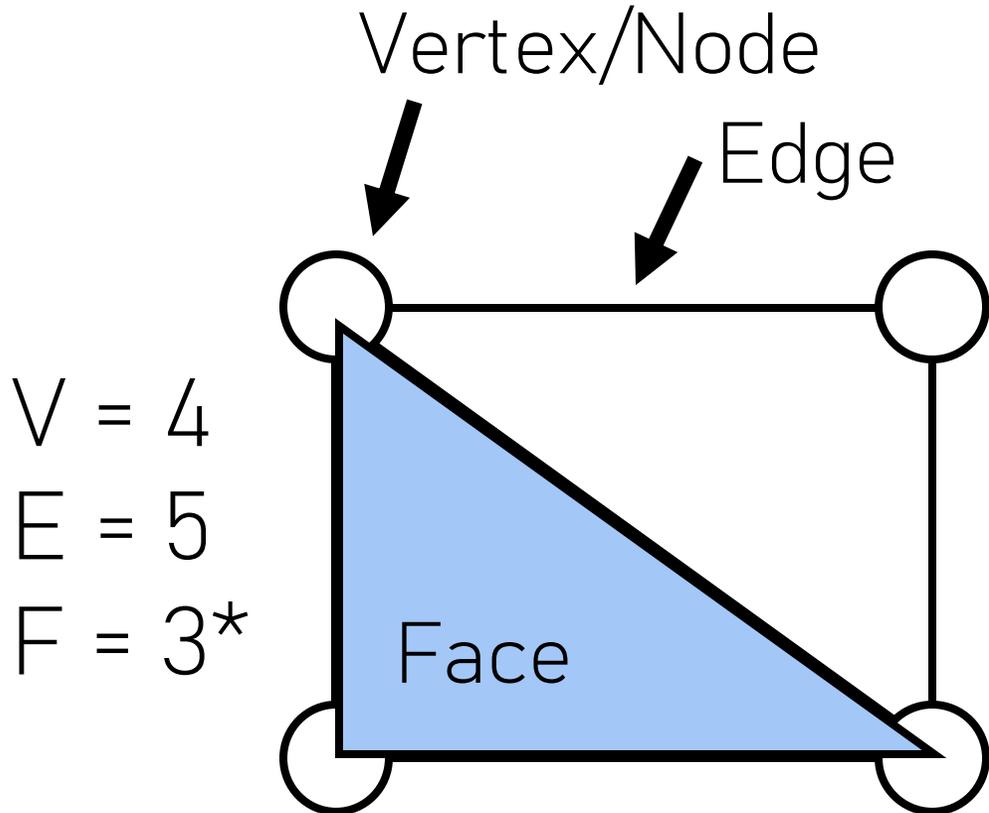


Graph Coloring Puzzle

Find a coloring of the following graphs using as few colors as possible.



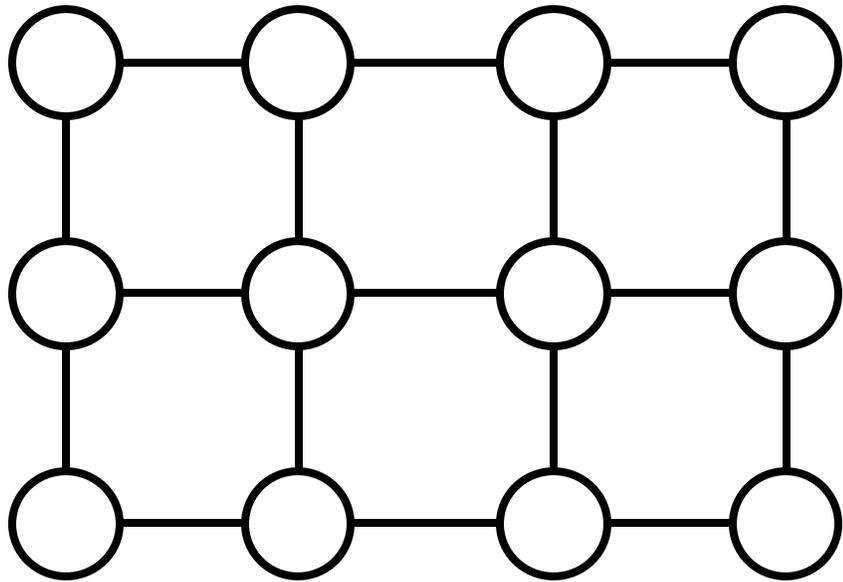
Euler's Formula for Planar Graphs



- A graph is **planar** if it can be drawn without crossing edges.
- What is the relationship between the parts?
 - V = number of vertices
 - E = number of edges
 - F = number of faces (including outside*)

$$V - E + F = ? \quad 4 - 5 + 3 = 2$$

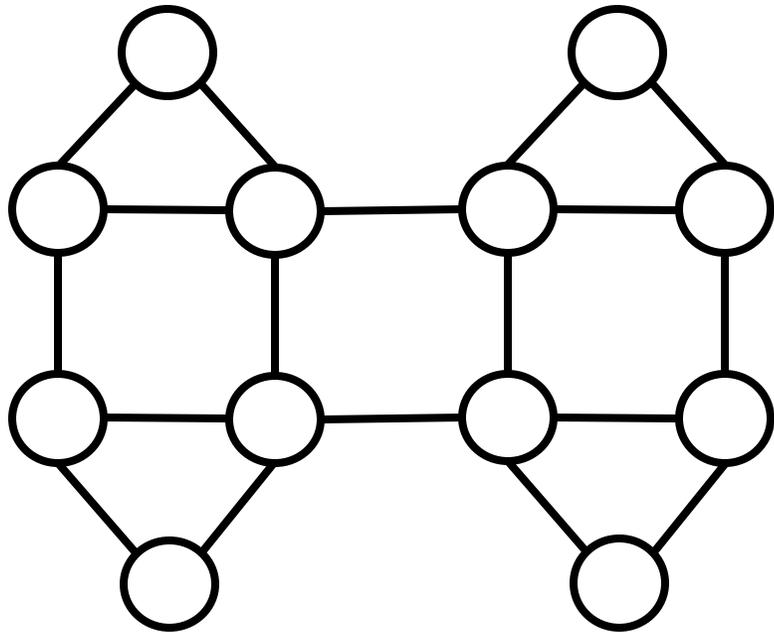
Graph 1:



$$V - E + F = ?$$

$$12 - 17 + 7 = 2$$

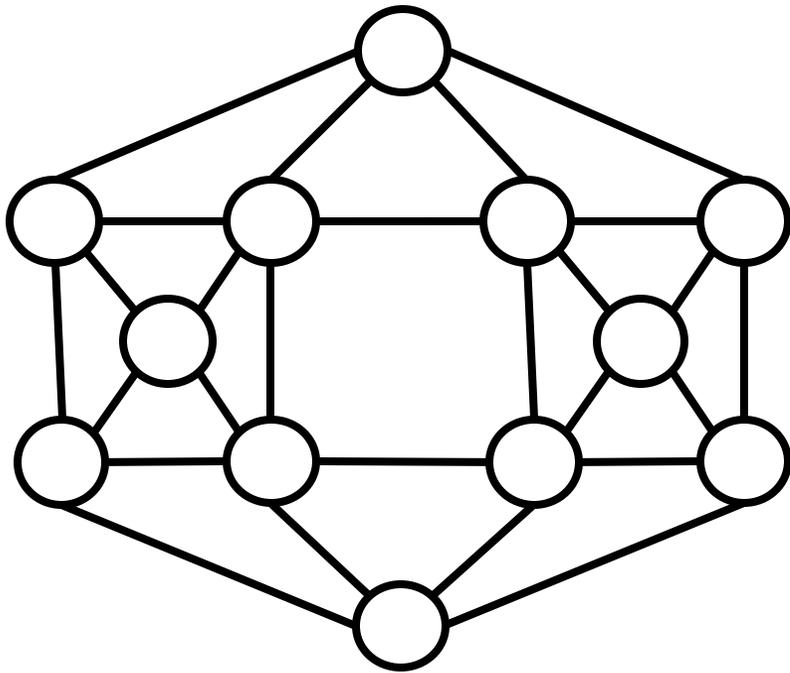
Graph 2:



$$V - E + F = ?$$

$$12 - 18 + 8 = 2$$

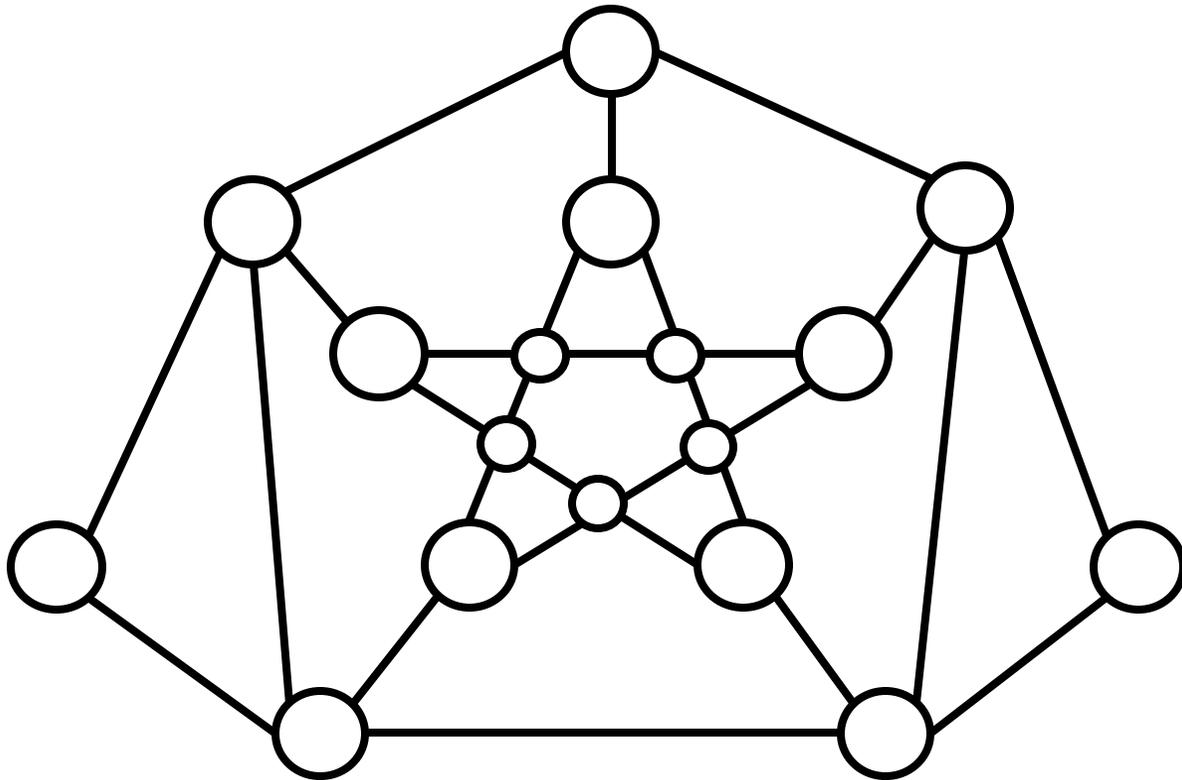
Graph 3:



$$V - E + F = ?$$

$$12 - 26 + 16 = 2$$

Graph 4:



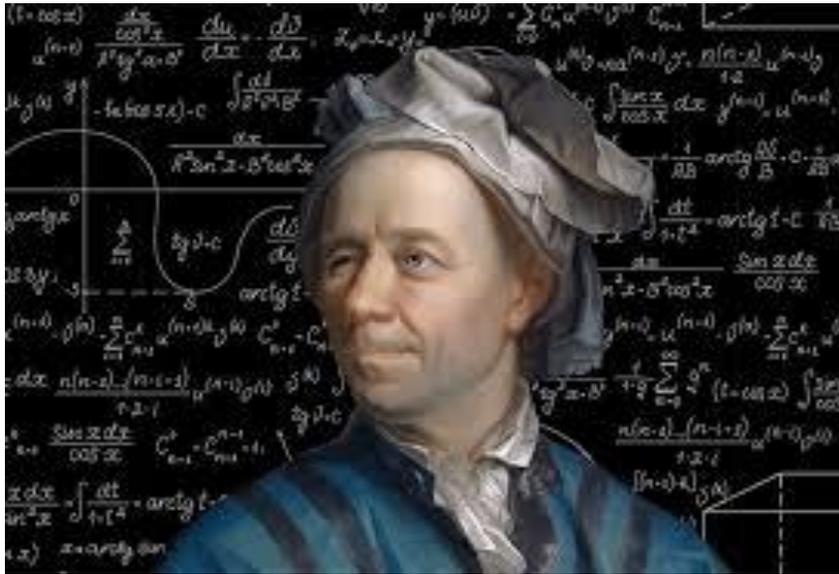
$$V - E + F = ?$$

What's wrong?

$$12 - 19 + ? = ?$$

$$17 - 29 + 14 = 2$$

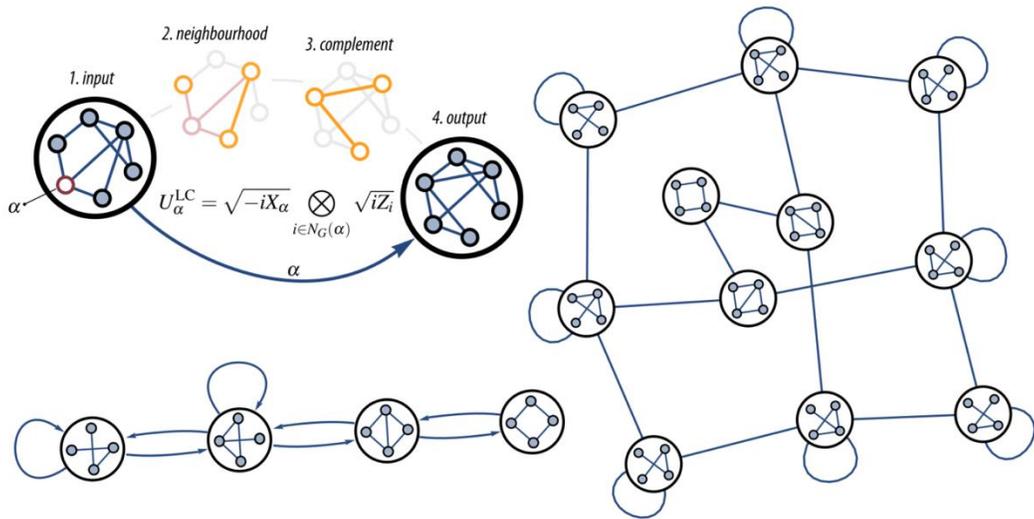
Euler's Formula for *Planar* Graphs



$$V - E + F = 2$$

Euler proved that all *planar* graphs obey this formula!

Examples of Graphs in Real Research!



Jeremy C Adcock, Sam Morley-Short, Axel Dahlberg, and Joshua W Silverstone. Mapping graph state orbits under local complementation. *Quantum*, 4:305, 2020.

