A Database of Tangles: Knot Theoretic Models for DNA Topology

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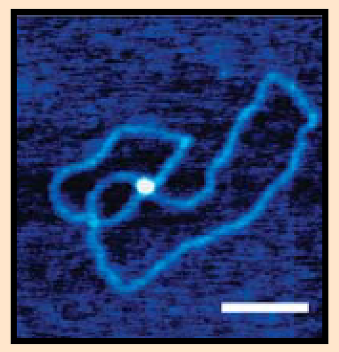
Application to DNA Topology



Motivation from Biology:
Molecular interactions involving DNA can be modeled using knot theory.

Figure 1: Protein DNA complex (AFM).

Vetcher, Alexandre A., et al. "DNA topology and geometry in Flp and Cre recombination." Journal of molecular biology 357.4 (2006).

Solution from Mathematics:
These interactions describe a system of tangle equations modeling entwined strings of DNA.

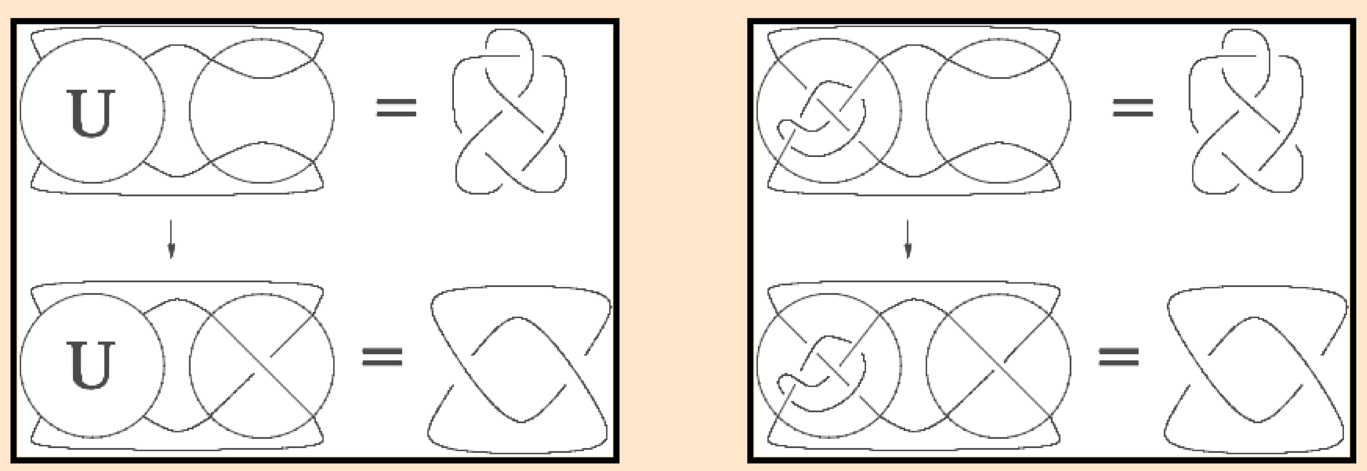


Figure 2: Example of a system of tangle equations with solution.

A **tangle** consists of multiple entwined strings embedded inside of a ball. They are best interpreted as the building blocks of knots.

Database of 2-String Tangles

Prototype database of tangles is accessible at `http://www.nick-connolly.com/tangles`.

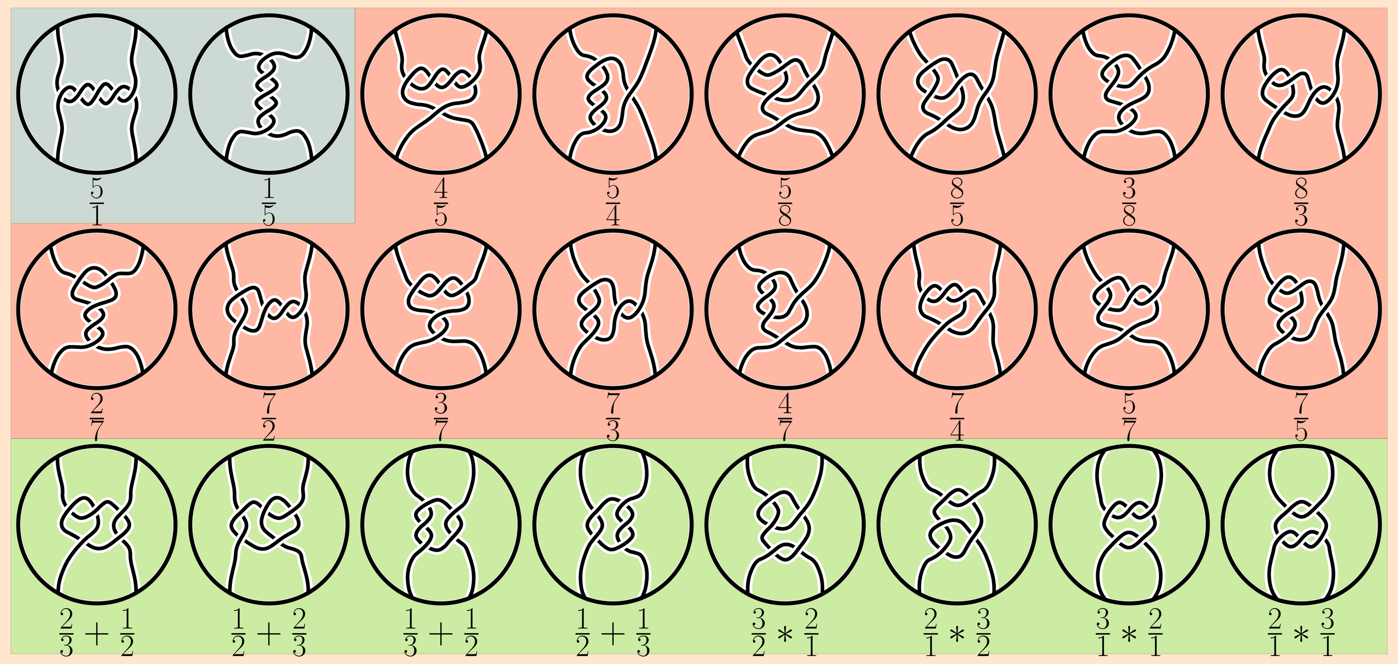


Figure 3: Table of 5 crossing tangles, excluding mirror images. Blue denotes integer, red for rational, and green for generalized Montesinos.

Tangle Algebra

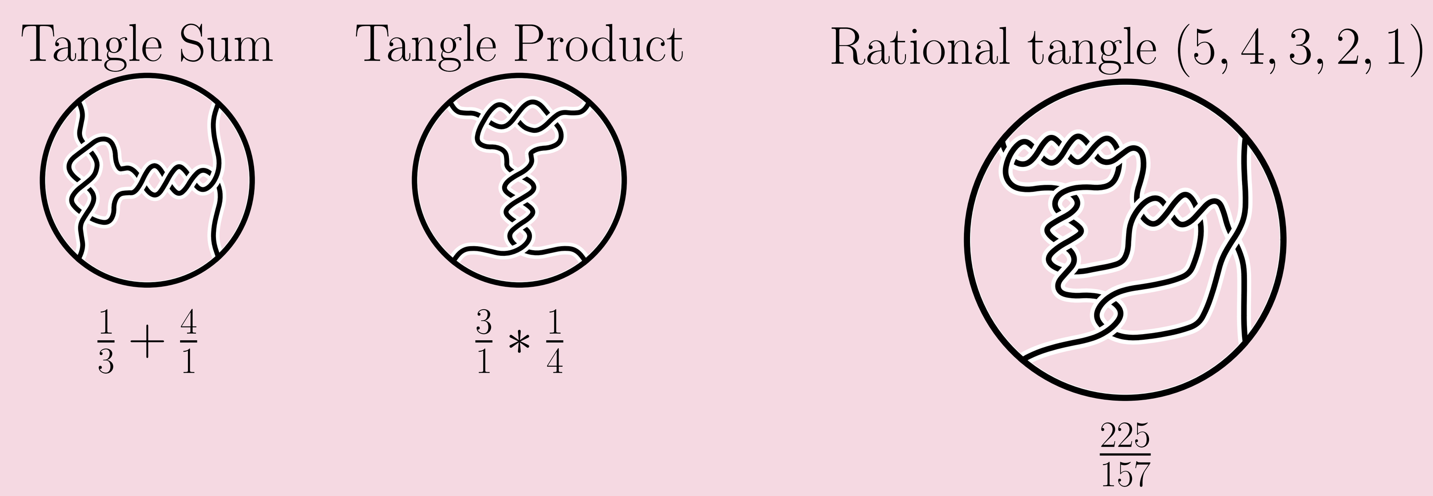


Figure 4: Operations of tangle sum (horizontal join) and tangle product (vertical join). Rational tangles are built using sums and products of integer tangles, described by a twist vector $(n_1, \cdots, n_k)$.

Simple tangles can be stacked together like blocks to build more complicated diagrams (Figure 4). Rational tangles, described by a twist vector $(n_1, \cdots, n_k)$, are in bijection with $\mathbb{Q} \cup \{\infty\}$ via the continued fraction

$$n_k + \cfrac{1}{n_{k-1} + \cdots + \frac{1}{n_1}} = \frac{p}{q}.$$

Tangles are divided into families based on their structure.

Special Types of Tangles

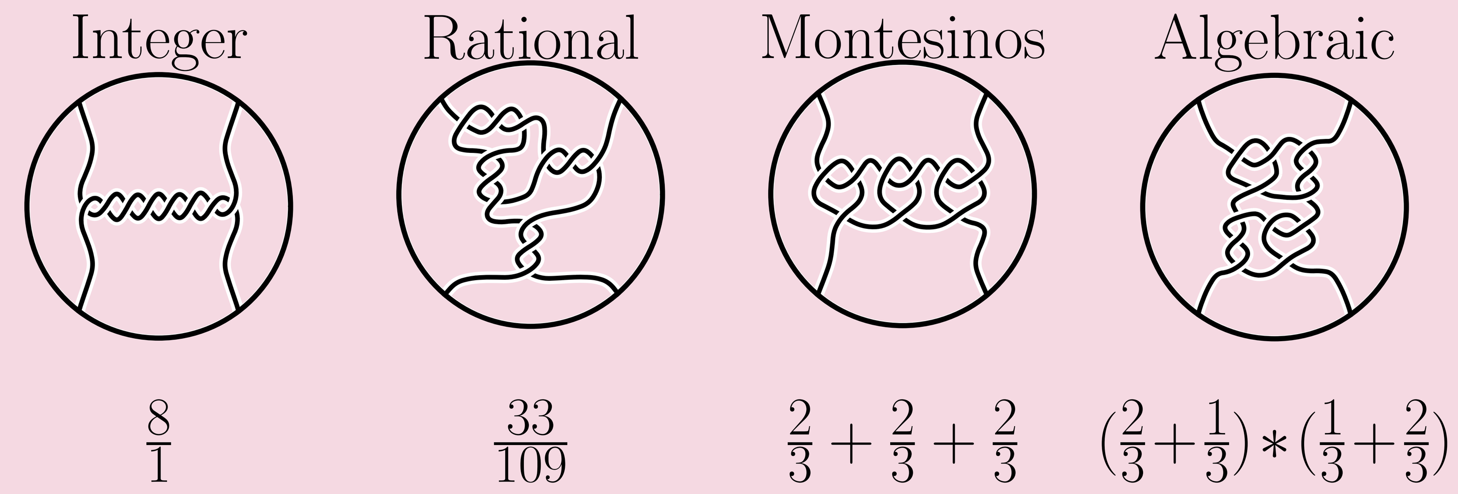


Figure 5: Examples of tangle families by increasing generalizability.

- An **integer** tangle is a horizontal sequence of twists.
- A **rational** tangle is constructed from a sequence of alternately horizontal and vertical twists $(n_1, \cdots, n_k)$.
- A **Montesinos** tangle is a sum of rational tangles.
- An **algebraic** tangle is constructed from any combination of sums and products of rational tangles.

$$\{\text{Integer}\} \subseteq \{\text{Rational}\} \subseteq \{\text{Montesinos}\} \subseteq \{\text{Algebraic}\}$$

- Otherwise, a tangle is **non-algebraic**.

Subtangle Decompositions

Subtangle Planar Diagram Code

(2/3)	0 NW	2 NW	5 NW	4 SE	(P : 0′)
(3/2) ∗ (2/1)	1 NE	0 NE	3 SW	5 NE	(P : 0)
(4/1)	0 SE	4 NW	5 SE	2 SE	(P : 0)
(1/3)	3 NE	0 SW	1 SW	5 SW	(P : 1′)
(3/7)	1 SE	2 SW	3 SE	4 SW	(P : 1)

Constellation Notation

$$C_{5+1}\Big(\left(\tfrac{2}{3}\right), \left(\tfrac{3}{2} * \tfrac{2}{1}\right), \left(\tfrac{4}{1}\right), \left(\tfrac{1}{3}\right), \left(\tfrac{3}{7}\right) \Big)$$

Non-Algebraic Diagram Decomposition

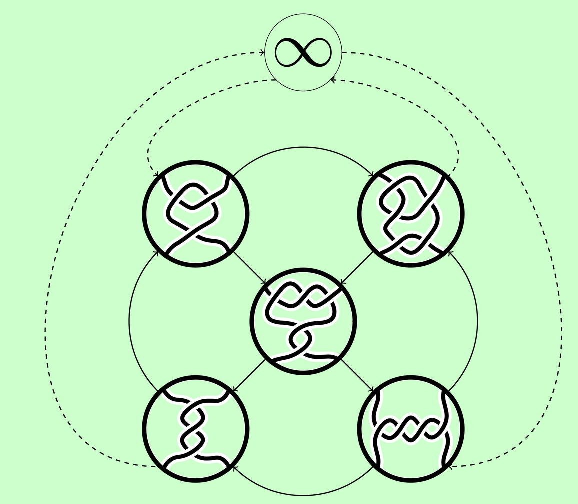


Figure 6: Example of a subtangle decomposition for a non-algebraic diagram and corresponding notations. This configuration of subtangles matches the first constellation graph of Figure 7.

Non-algebraic tangles can be decomposed into algebraic subtangles (Figure 6). This decomposition is described by a **subtangle planar diagram** notation encoding the adjacency list of a graph. Non-algebraic subtangle graphs have special properties and are known as constellations.

Constellations

A $k+1$ **constellation** (C, p) consists of a graph C with k vertices and a special marked $k+1^{\text{st}}$ vertex such that C is connected, planar, 4-valent, and contains no bigons.

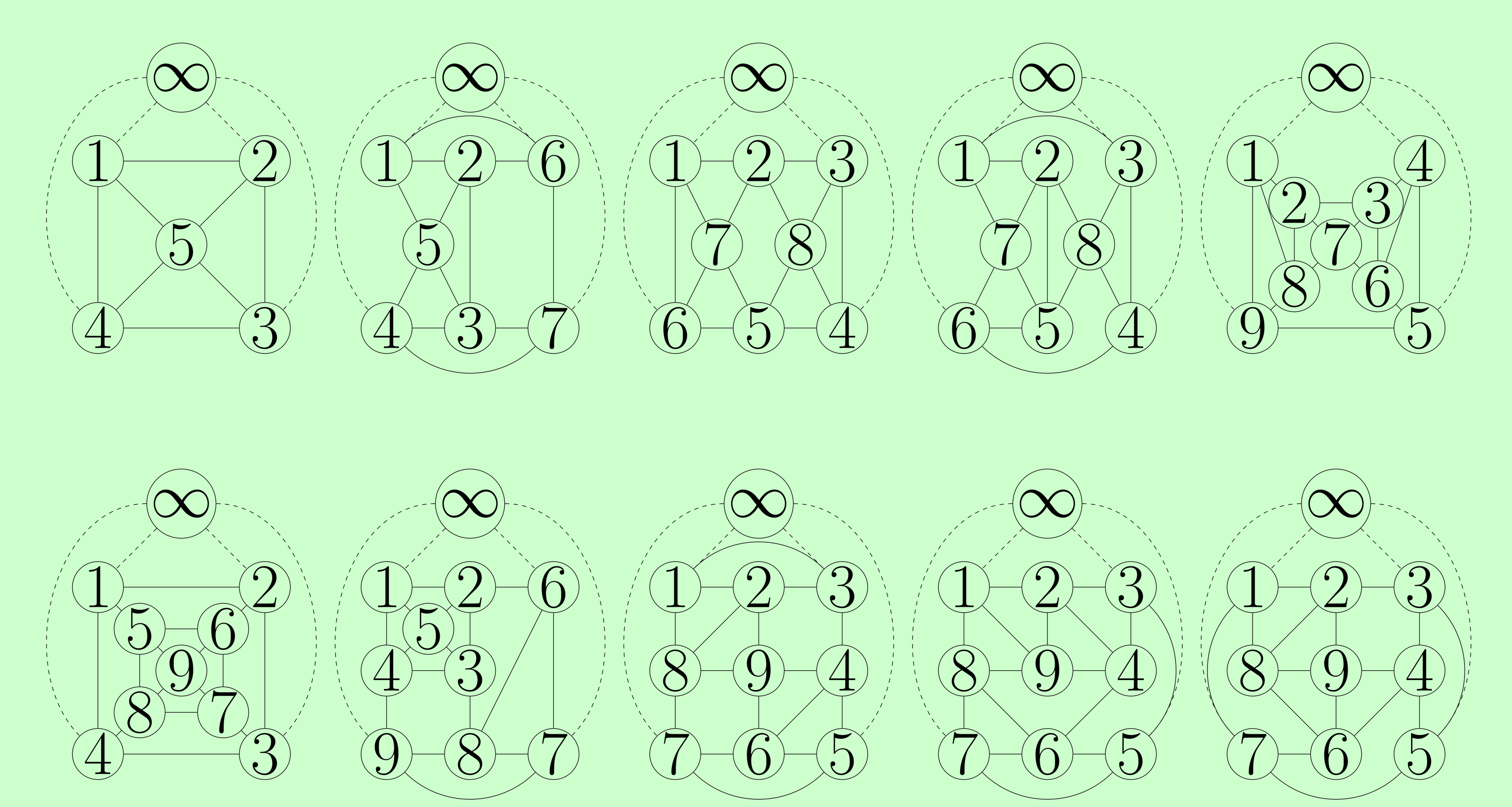


Figure 7: The first 10 constellation graphs for constructing non-algebraic diagrams. With sums and products, these can be used to describe all non-algebraic tangle diagrams with at most 9 crossings.